Improved Predictions of Confined Jets with a Parabolic Computation of the Entrance Region

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Introduction

IN the course of a numerical study of confined turbulent jets with the k-ϵ model, it was found that the upstream conditions prescribed for the turbulence had a considerable influence on the predictions. The purpose of this Note is to propose a rational scheme to specify these boundary conditions for elliptic flow computations and to demonstrate the improvements produced by the proposed scheme.

The flow under consideration (sketched in Fig. 1) is elliptic because of the existence of an adverse pressure gradient that eventually leads to recirculation. The main features of the flow are controlled by two factors: 1) the inlet Craya–Curtet number[1] $C_t = 1/\sqrt{m}$ where m is the total impulse nondimensionalized by the total flow rate and duct cross-sectorial area, all three quantities being determined at the inlet, and 2) the geometry of the duct. Recirculation occurs in cylindrical ducts[1] for $C_t \leq 0.9$ and in the conical duct[2] with a divergence angle $\theta = 2.5$ deg for $C_t \leq 1.1$.

When low turbulence is prescribed at the upstream boundary, close agreement of the predictions with the experimental data can be obtained only when the computation is started at a distance $x_i = 3.75d_0$ from the inlet. This is illustrated by the decay of the jet centerline velocity shown in Fig. 2. Since there is no physical reason for such a shift of the upstream boundary and since the optimal value of x_i is purely empirical, this procedure is not very satisfactory.

Another procedure involves prescribing a length-scale distribution at the entrance section, as

$$L = 0.038d_0 \qquad r < d_0/2$$

$$L = [0.07 - 0.04(2r/D_0)^2 - 0.03(2r/D_0)^4]D_0 \qquad r \geq d_0/2$$

The second formula is the Nikuradse[3] distribution for fully developed pipe flow. With L thus specified, k and ϵ are computed from the mixing length and equilibrium assumptions as in the procedure developed in the next section. The results obtained with this scheme correspond to the curves labeled L at $x_i = 0$ in Fig. 3. Figure 3a applies to the same cylindrical geometry as Fig. 2 and Fig. 3b to the conical duct with $\theta = 2.5$ deg.[2] It is clear that the decay of the computed centerline velocity does not compare well with the experimental data in either case.

Thus, neither of these two schemes is satisfactory and another approach is needed for the specification of the upstream boundary conditions of the turbulent quantities.

Parabolic Entrance Region Scheme

In the geometries investigated, the ratio D_0/d_0 is large (13 and 10, respectively), so that the initial shear layers of the jet are quite thin compared to the duct radius. If the number of grid points is kept within reasonable values in the elliptic code, then the grid is too coarse in the initial jet shear layers. Moreover, since $\epsilon = k^{3/2}/L$ and L is proportional to the shear layer thickness, ϵ may be very large and this, combined with the coarseness of the grid, could well account for the poor results of the earlier computations.

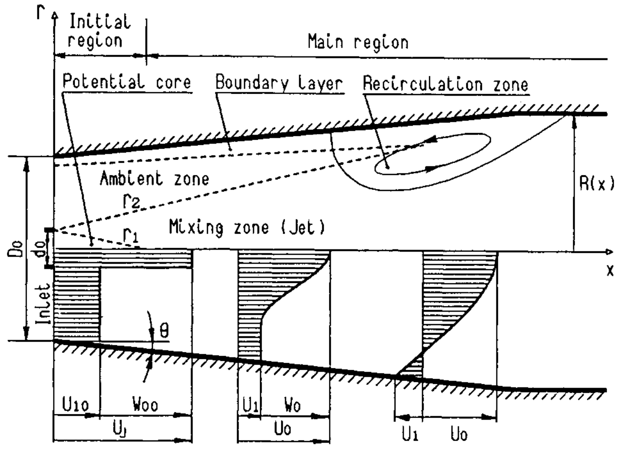

Fig. 1 Flow configuration and definitions.

A solution to the aforementioned difficulties might be a parabolic computation of the entrance region. This is similar in spirit to the parabolic sublayer (PSL) scheme proposed by Iacovides and Launder[4] for a fine-grid treatment of near-wall flow. Such a parabolic computation is justified by the fact that although the flow as a whole must be regarded as elliptic, there is a short parabolic region near the entrance. In this region, the flow is predominantly axial and independent of the pressure field further downstream where the recirculation occurs.

The proposed parabolic entrance region (PER) scheme consists of two steps: 1) a parabolic computation over a short entrance region between $x = 0$ and $x = x_i$; and 2) an elliptic computation with the upstream boundary shifted to x_i using the results of the parabolic computation as boundary conditions.

This scheme is applied to the two confined jets considered previously. The standard k-ϵ model and the mixing length model are used for the computation of the parabolic flow. For the latter,

$$\overline{uv} = -\nu_t \frac{\partial U}{\partial r} \qquad \nu_t = L^2 \left| \frac{\partial U}{\partial r} \right| \qquad L = C(r_2 - r_1)$$

where r_2 and r_1 are the coordinates of the outer and inner edge of the jet shear layer and C an empirical coefficient determined form the experimental data of Rajaratnam and Pani,[5]

$$C^2 = 0.0042 + 0.004(U_{10}/U_j) \qquad 0 \leq U_{10}/U_j \leq 0.2$$

The values of r_2 and r_1 at $x = 0$ are inferred from the confined jet data and are computed between $x = 0$ and x_i. The starting values for k and ϵ are obtained from the value of $\overline{uv}$ calculated with the preceding relations and from the equilibrium assumption such that

$$k = -\overline{uv}/\sqrt{C_\mu} \qquad \epsilon = C_\mu k^2/\nu_t$$

For the computation of the parabolic flow, the explicit finite-difference scheme of Pletcher[6] is used with grid spacings $\Delta r = d_0/40$ and $\Delta x = r/10$. The computation of the ellip-

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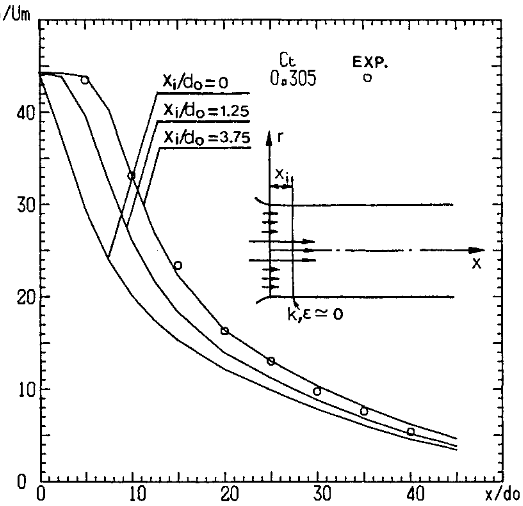

Fig. 2 Jet centerline velocity decay.

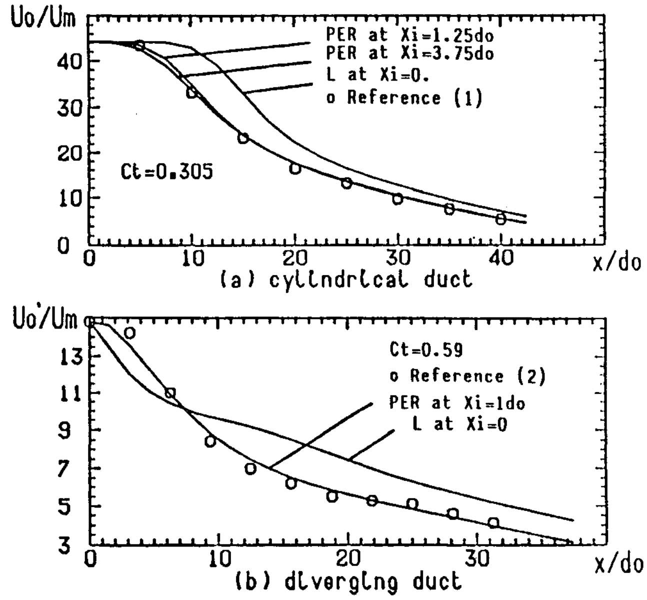

Fig. 3 Jet centerline velocity decay.

tic flow is carried out with the solution algorithm of Benodekar et al.[7]

Exploratory tests are conducted for several mesh sizes: 22×16, 32×20, 42×24, and 52×34. The results with the last two grid systems are essentially the same. Thus, the 42×24 computation can be considered to be grid independent. Further details of the numerical methods employed are given in Ref. 8.

The results concerning U, $\overline{uv}$, r_1, and r_2 obtained with the mixing length model in the parabolic computation are in close agreement with those obtained with the k-ϵ model and with the available experimental results.[8] The simpler model is therefore adequate for this computation.

The performance of the PER scheme for the two confined jets is illustrated in Fig. 3. It is seen that good agreement with the experimental results is obtained if the parabolic computation is carried out over a distance $x_i \simeq 1d_0$; there is only a slight change if this computation is extended to $3d_0$. Particularly remarkable is the improvement of predictions for the diverging duct where the other upstream boundary conditions predicted a curve of a different shape. The time required for the parabolic computation is negligible compared to that of the elliptic flow (less than 1%).

Conclusion

It is shown that the PER scheme, based on a parabolic computation of the upstream boundary condition for an elliptic flow, leads to notable improvements in the prediction of confined jets. The additional computational effort is minimal and no arbitrary adjustments are required. A simple mixing length model may be adequate for this purpose. The results presented here encourage the application of this scheme to other flows presenting similar difficulties.

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Rayleigh Measurements of Species Concentration in a Complex Turbulent Flow

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Introduction

SOLID-FUELED ramjets (SFRJ) require a flame-anchoring region at the head of the combustion chamber, as do other ramjets. This low-velocity region in which air and pyrolyzed fuel are allowed to mix may be achieved by generating a recirculation zone behind a backward-facing step. The laboratory at Georgia Tech is currently investigating the complex flowfield over a backward-facing step with a bleed flow from the floor behind the step, for both cold flow and flow with combustion. A schematic of this SFRJ simulator is shown in Fig. 1. The bleed flow from the lower wall simulates the gases emanating from a solid fuel. As previously reported,[1] mean velocities, turbulence intensities, and shear stresses have been calculated using a modified k-ϵ model and measured using a laser Doppler velocimeter (LDV) for the cold flow. Also calculated were the injectant mass fraction distributions in the cold

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